Exercise 8

Calculate y'.

$$xe^y = y\sin x$$

Solution

Take the derivative of both sides with respect to x.

$$\frac{d}{dx}(xe^y) = \frac{d}{dx}(y\sin x)$$

$$\left[\frac{d}{dx}(x)\right]e^y + x\left[\frac{d}{dx}(e^y)\right] = \left[\frac{d}{dx}(y)\right]\sin x + y\left[\frac{d}{dx}(\sin x)\right]$$

$$(1)e^y + x\left[(e^y)\cdot\frac{d}{dx}(y)\right] = \left[\frac{d}{dx}(y)\right]\sin x + y(\cos x)$$

$$e^y + xe^y\frac{dy}{dx} = \frac{dy}{dx}\sin x + y\cos x$$

Solve for dy/dx.

$$xe^y\frac{dy}{dx} - \frac{dy}{dx}\sin x = -e^y + y\cos x$$

$$(xe^y - \sin x)\frac{dy}{dx} = -e^y + y\cos x$$

As a result,

$$\begin{aligned}\frac{dy}{dx} &= \frac{-e^y + y\cos x}{xe^y - \sin x} \\ &= \frac{-\left(\frac{y\sin x}{x}\right) + y\cos x}{(y\sin x) - \sin x} \\ &= \frac{y\left(\cos x - \frac{\sin x}{x}\right)}{(y-1)\sin x}\times\frac{x}{x} \\ &= \frac{y(x\cos x - \sin x)}{x(y-1)\sin x}.\end{aligned}$$